ACTION OF WIND GUSTS AND WEAK SHOCKS ON AN AIRFOIL IN A TRANSONIC STREAM

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We consider the problem of the interaction of unsteady disturbances (wind gusts and low-intensity shocks) with an airfoil in a transonic stream of ideal gas. We investigate the dynamics of compression shocks that enclose local supersonic zones and determine the influence of the movement of shocks on the unsteady aerodynamic characteristics. We analyze the influence of the nonlinearity of the problem associated with a transonic regime of motion. We give the results of numerical calculations of the problem for specific cases of the action of aperiodic disturbances: horizontal wind gusts and shocks that overtake the airfoil.

Much interest has recently been devoted to the investigation of unsteady transonic flows [1]. This is important, for example, for the aerodynamics of helicopters [2-4], acoustics problems, and aeroelasticity problems [5-7], since small disturbances of stream parameters can significantly affect the positions and intensities of shocks, which, in turn, strongly affect the integrated aerodynamic characteristics. Most of the earlier research on the interaction of unsteady disturbances with bodies has involved disturbances of a periodic nature [2, 3, 5, 8], which makes it possible, generally speaking, to simplify the problem, using the so-called low-frequency approximation. In the analysis of certain cases (vibration of a flap, for example), one can then represent the unsteady solution as a combination of a steady nonlinear solution and an unsteady linear one [2].

The action of aperiodic disturbances on a body in a transonic stream presents a more complicated problem. We note [4], in which the interaction of aperiodic disturbances caused by the motion of vortices with an airfoil was studied within the framework of a nonlinear transonic theory.

In the present paper we analyze the action of aperiodic disturbances on an airfoil moving in a transonic stream. As the disturbances we consider wind gusts and moving weak shocks.

1. Statement of the Problem. Let a steady transonic stream of ideal gas flow with velocity U_∞ over a thin airfoil set at a small angle of attack. At the initial time an unsteady disturbance develops in the stream in the form of a wind gust, which instantaneously encompasses the airfoil, or a weak shock, whose front is initially located a certain distance from the body. It is required to investigate the unsteady process of flow over the airfoil and the change in the airfoil's integrated aerodynamic characteristics in the transitional regime.

Since the Mach number of the oncoming stream is $M_\infty \approx 1$, while the thicknesses of the airfoils under consideration, the gust velocities, and the shock intensity (the pressure drop at the shock front) are small compared to the characteristic values of the analogous quantities in the problem, we can use the transonic theory of small disturbances. In the context of that theory, the problem is described by a nonlinear unsteady equation for the potential φ of the disturbed velocity [3, 4]:

$$B\varphi_{tt} + 2B\varphi_{xt} = \frac{\partial}{\partial x}\{[C_1 + C_2(\varphi_x + u_G)](\varphi_x + u_G) - C_1 u_G\} + \varphi_{yy}. \quad (1.1)$$

Here $B = M_\infty^2$; $C_1 = 1 - M_\infty^2$; $C_2 = -(\gamma + 1)M_\infty^m/2$; γ is the adiabatic index; u_G is the horizontal velocity component of the external disturbance.

We place the origin O of the Cartesian coordinate system at the center of the chord of the airfoil, and we direct the Ox axis along the velocity vector of the undisturbed oncoming stream and the Oy axis upward, perpendicular to the Ox axis. We assume that all quantities in Eq. (1.1) and below are normalized to their characteristic values:

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$$x_0 = y_0 = l,\ u_0 = U_\infty,\ t_0 = x_0/u_0,\ \varphi_0 = u_0 x_0$$

(ℓ is the length of the chord of the airfoil).

We represent the velocity field in the form of three components: the velocity $\mathbf{u}_\infty$ of the uniform oncoming stream, the velocity $\mathbf{v}_G$ of the external disturbance, and the gradient $\nabla\varphi$ of the unknown disturbance of the velocity potential, i.e.,

$$\mathbf{v} = \mathbf{u}_\infty + \mathbf{v}_G + \nabla\varphi. \tag{1.2}$$

Using Eq. (1.2) does not mean that the problem has been linearized, since Eq. (1.1) is nonlinear and it cannot be solved in the form of the corresponding superposition.

Let the contour of the airfoil be described by the equation $y_b = f(x, t)$; we then write the boundary condition of nonpenetration at the airfoil as

$$\left.\frac{\partial\varphi}{\partial y}\right|_{y=0} = \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) y_b - v_G,\ |x| < 0{,}5 \tag{1.3}$$

(v_G is the vertical velocity component of the external disturbance).

To obtain good agreement between solutions found within the framework of the transonic theory of small disturbances and using the Euler equations, we use a variable value of the exponent m in the expression for C_2. It is chosen for the specific application and depends on the solution itself in a certain sense [2]. In the present paper we use the procedure suggested in [9]: the exponent m is taken to be 1.75, and the inclination of the surface of the airfoil in the boundary condition (1.3) is divided by $M_\infty^{1/4}$. This increases the calculation accuracy due to compensation for errors introduced in the approximation of the boundary conditions at the line y = 0. The pressure coefficient c_p in the unsteady stream is calculated from the formula

$$c_p = -2(\varphi_x + u_G + \varphi_t). \tag{1.4}$$

The Chaplygin–Zhukovskii condition at the sharp trailing edge of the airfoil, when continuity of the vertical velocity and pressure in the wake – located at the Ox axis (y = 0, x > 0.5) in the approximation under consideration – is satisfied, leads to the relationship

$$(\Gamma_x + \Gamma_t)_{y=0} = 0 \quad (\Gamma = \varphi(x, -0, t) - \varphi(x, +0, t)). \tag{1.5}$$

Since in a numerical realization we solve the problem in a large but finite domain, we must eliminate the possible influence of the boundaries on the flow field due to reflection back into the stream of disturbances that reach them. We achieve this by applying nonreflective boundary conditions, developed for Eq. (1.1) in [10]. Disturbances from the body that reach the boundaries of the calculation domain then pass freely through them.

Equations (1.1)-(1.5), together with the aforementioned conditions at the boundaries of the calculation domain, completely describe the problem of unsteady interaction of an airfoil with (instantaneously encompassing) wind gusts in a transonic stream.

In modeling the action of weak shocks on an airfoil, the above method is inapplicable because the velocity field of the external disturbances is not known in advance, since both the shock's intensity and the position of its front change in the process of the interaction. We therefore use another approach. In Eqs. (1.1)-(1.5) we take $u_G = v_G = 0$, and the shock is modeled by specifying a certain potential distribution at a fairly large distance from the airfoil, where the stream is hardly disturbed. The weak shock, given by the potential distribution at the time t = 0, starts to propagate at t > 0 in accordance with gas-dynamic laws in the field of the disturbed flow profile.

2. Method of Solution. We solve the problem numerically using an implicit finite-difference method of variable directions [8], using the Engquist–Osher algorithm [11] for spatial differentiation, depending on the type of equation. This algorithm is monotonic and automatically eliminates nonphysical solutions in the form of rarefaction shocks. In contrast to explicit methods, there are no limits on the time step associated with the analysis of stability, which enables us to consider the entire transition process up to the new steady state with no significant increase in calculation time. In the investigation of the unsteady solution, however, Δt must not exceed the time in which the shocks in the flow field travel a distance greater than one cell of the calculation grid. Test calculations show that in the analysis of transitional processes in the transonic range, calculations must be carried out with sufficiently small Δt (0.01-0.04) in the initial calculation stage for a correct description of the dynamics of shocks that enclose the supersonic domain, especially when they lie near the trailing edge.

The calculation scheme is conservative. Compression shocks are not specially singled out, but are obtained in the process of calculation in the form of a strong bunching of isobars in the flow field. The solution process is not iterative, but φ^{n+1} at the (n + 1)-th time layer is found directly from φ^n at the n-th layer.

Numerical calculations of the problem of the action of unsteady wind gusts were carried out on a grid of 91 nodes in the x direction and 60 nodes in the y direction. To better reveal the dynamics of shocks in the stream in the problem of the action on an airfoil of a shock overtaking it from behind, the number of nodes was increased to 167 in x and 120 in y. The grid is bunched in the regions of the leading and trailing edges and spread out toward the outer boundaries of the calculation domain, which lie at distances of 80 chord lengths from the body in the x direction and 120 chord lengths in the y direction. At the airfoil, which lies on the x axis ($|x| < 0.5$) there are 55 nodes.

The solution of the problem of steady transonic flow over the airfoil, needed as an initial condition, is also obtained using the present algorithm by the establishment method. The steady-state solutions thus found agree well with solutions of the Euler equations and the complete potential found by other methods.

3. Action of Wind Gusts. The problem of the interaction of an airfoil with a horizontal wind gust in the opposite direction from that of the airfoil is of great interest from the practical standpoint, and for revealing features that specifically characterize the transonic region. It is important to investigate the dynamics of shocks in the transitional process, since they cause considerable changes in the integrated aerodynamic characteristics.

In Fig. 1 we give values of the lift coefficient $c_y(t)$ as a function of time for cases in which horizontal gusts, in the direction opposite to that of the airfoil, with velocities $u_G = 0.037$, 0.074, 0.147, and 0.221 (curves 1-4), act on a biconvex airfoil formed by arcs of a circle, with a thickness $\delta = 0.04$, at an angle of attack $\alpha = 2°$, and in a stream with $M_\infty = 0.8$. It should be noted that these u_G correspond to actual wind gusts of 10, 20, 40, and 60 m/sec observed in nature.

The general scheme of the interaction for this example is as follows. The horizontal gust causes an increase in the velocity of the oncoming stream. The pressure curve at the surface of the airfoil "fills out." The stream velocity is considerably higher at the upper surface than at the lower surface, and at some time after the gust starts to act, a large supersonic zone forms at the upper surface. The shock enclosing the supersonic zone gradually shifts toward the trailing edge, and its intensity increases simultaneously. If a supersonic zone is also formed at the lower surface, then the shock enclosing it also starts to move toward the trailing edge, but slower than the shock at the upper surface, since the stream velocity is lower at the lower surface. If there are no supersonic zones, then the areas under the pressure curves increase, and faster at the upper surface. As a result, an increase in lift is observed in the initial stage of the interaction.

If the gust is sufficiently strong, then the shock at the upper surface already reaches the trailing edge in the transitional process, and it does not move farther. At the same time, the downstream movement of the shock at the lower surface continues. This results in a decrease in the difference between the areas of the pressure curves and a consequent decrease in lift. The characteristic swing in the lift in the transitional period (solid curves 3 and 4 in Fig. 1) is formed in this way.

Depending on the conditions of the original streamline flow and on the magnitude of the gust, the new steady-state value of c_y may be either larger or smaller than the initial value. As seen from Fig. 1, a large gust can result in a lower new steady-state value of c_y. This is because the steady-state lift coefficient as a function of Mach number in the near-sonic range has a maximum; the initial and final points of the transitional process lie on different sides of the maximum on that curve.

The nonmonotonic behavior of the aerodynamic characteristics in the transitional period reflects a fundamental difference between the nonlinear and linear problems; the transition to the new regime is monotonic in the linear problem. The dashed curves in Fig. 1 give the numerical solutions of this problem in the linear formulation. It is obtained by setting $C_2 = 0$ in Eq. (1.1). The nonlinearity is manifested, as seen from Fig. 1, in a slowing of the transition to the new steady streamline flow, in addition to the property indicated above. Moreover, the new linear steady-state value differs considerably from that obtained in the nonlinear theory, and that difference increases for larger gusts.

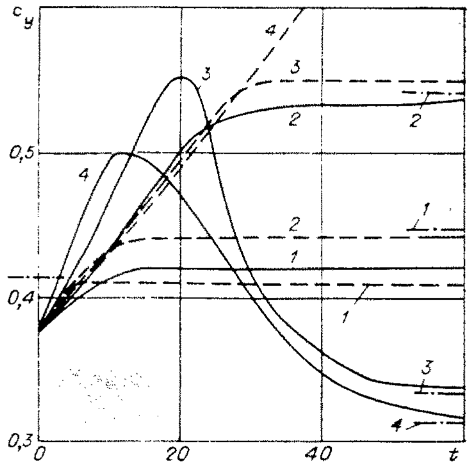

Fig. 1

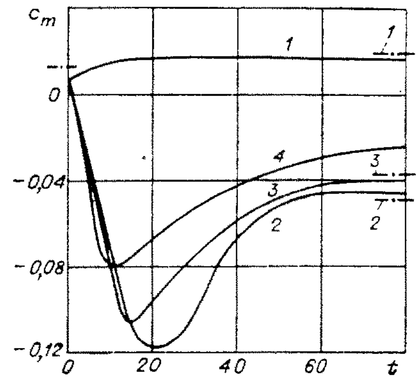

Fig. 2

The latter conclusions are consistent with the results of [4, 8], in which the problems of a sudden vertical downdraft and of entry into a vertical gust were considered. We also note that the swing in the c_y curve observed in the interaction of horizontal gusts with an airfoil can be several times larger than the steady-state value of the lift coefficient.

The behavior of the torque coefficient relative to 1/4 of the chord, shown in Fig. 2, also displays nonmonotonic properties associated with features of transonic streamline flow (the notation here is analogous to that in Fig. 1). The movements of shocks over the airfoil have a stronger influence on the variation of c_m than c_y, since the moment arm of the applied torque varies along with the area of the pressure curves on the surface of the airfoil.

The horizontal dot-dashed lines in Figs. 1 and 2 represent the steady-state values of c_y and c_m for the given airfoil, obtained experimentally in [12], that correspond to the flows established after the action of the gusts. Good agreement between the experimental and numerical results is observed.

Conclusions analogous to those described above are also obtained in the analysis of problems of the unsteady interaction of airfoils with vertical wind gusts in a transonic stream. Moreover, a comparison of the nonlinear numerical solutions with linear numerical solutions obtained by the same method and with the well-known analytical linear solutions of [13, 14] enables us to conclude that the influence of nonlinear terms is stronger in the subsonic range than at $M_\infty > 1$.

4. Unsteady Interaction of a Weak Shock with an Airfoil. To determine the efficacy of our approach to the study of the action of weak shocks on an airfoil, we investigate the interaction with a shock that overtakes the airfoil. This is one of the typical cases that brings out the features of unsteady interaction in a transonic stream.

In Fig. 3 we give, in the form of isobars, the evolution of the flow pattern in the interaction of a shock ($\Delta p = 0.21$) with an airfoil formed by arcs of parabolas, with a thickness $\delta = 0.1$, at an angle of attack $\alpha = 1°$, and in a transonic stream with $M_\infty = 0.875$.

At $t = 0.8$ (Fig. 3a), the wave has not yet reached the trailing edge of the airfoil. The flow ahead of it is undisturbed. The integrated aerodynamic characteristics have not changed and correspond to the initial steady-state values.

In Fig. 3b we show the time $t = 4.2$, when the shock has interacted with the shocks that enclose the supersonic zones and is moving over the airfoil. Since the stream velocity ahead of the shock is lower at the lower surface of the airfoil than at the upper surface, it travels faster there. The flow pattern does not change ahead of the shock; the stream flows with a new velocity over the part of the airfoil behind the shock.

The asymmetry in the passage of the shock over the upper and lower surfaces of the airfoil is even more clearly expressed in Fig. 3c ($t = 7.8$). At $t = 10.2$ (Fig. 3d) the wave at the lower surface passes the leading edge, curving and weakening near it, and departs upstream, while at the upper surface it is still on the body. The stream flows over most of the airfoil with the velocity corresponding to that behind the shock. The pressure curve at the lower surface of the airfoil gradually begins to "fill out." It should be noted that where

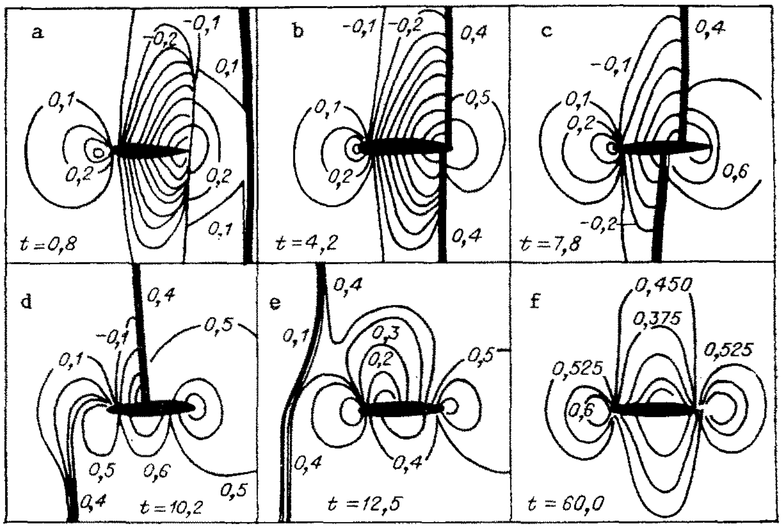

Fig. 3

the stream is weakly disturbed (two or three chord lengths up and down from the airfoil), the velocities of the shock fronts at the upper and lower surfaces are the same. In Fig. 3e (t = 12.5) we show the time when the shock at the upper surface has also passed the leading edge. The stream with the new velocity flows over the entire airfoil. The front of the shock departing upstream is curved. The pressure curves continue to "fill out."

Finally, in Fig. 3f (t = 60.0) we show the established flow corresponding to the stream with a new M_∞, lower than before. The case under consideration is characterized by the fact that subcritical streamline flow, i.e., with no supersonic zones at the body, is established as a result of the interaction. At the same time, when the intensity of the overtaking weak shock is lower than in the above example (we carried out calculations for shocks with pressure drops Δp = 0.05, 0.1, and 0.16 at the front), the supersonic zones do not completely disappear from the flow field. The time required for the complete establishment of the new regime in unsteady interaction in a transonic stream is very long and, as seen from Fig. 3, equals the time required for the airfoil to travel about 100 chord lengths.

In Fig. 4 we give the variation of the pressure coefficient over the chord of the airfoil at its lower and upper surfaces at the same times as in Fig. 3 (c_p^* is the critical value of the pressure coefficient). It is clearly seen that as the shock passes over the airfoil, the pressure hump is "eaten away," and this occurs faster at the lower surface (Fig. 4a-d). This results in an increase in the area between the pressure curves at the lower and upper surfaces, which increases the lift. As seen from Fig. 4d-f, the area between the pressure curves starts to decrease after the shock leaves the lower surface of the airfoil. The increase in lift becomes a decrease as the new steady-state value is reached. A swing thus occurs in the lift during the interaction.

The variation of the lift coefficient $c_y(t)$ during unsteady interactions of an airfoil with shocks of different intensities is given in Fig. 5 (curves 1-4 correspond to Δp = 0.05, 0.1, 0.16, and 0.21). The new steady-state value of c_y obtained as a result of the interaction is determined entirely by the initial conditions of the problem and can be either larger or smaller than the original lift.

All of the functions are represented so that the shocks reach the trailing edge at the same time. On the curve for Δp = 0.21 we plot the points corresponding to the times selected in Figs. 3 and 4. It should be noted that the swings of the curves can exceed severalfold the steady-state values of c_y, and since they are active in the flow for a fairly long time, their influence can be significant. The behavior of the torque $c_m(t)$ in these cases is also characterized by large swings and qualitatively duplicates the behavior of $c_y(t)$ in Fig. 5.

The observed nonmonotonic changes in the integrated aerodynamic characteristics of an airfoil with monotonic variation of the intensity of a shock that overtakes the airfoil, as in the case of the action of horizontal wind gusts, are characteristic of the transonic velocity range, indicating the necessity of investigating these problems only within the framework of the nonlinear theory.

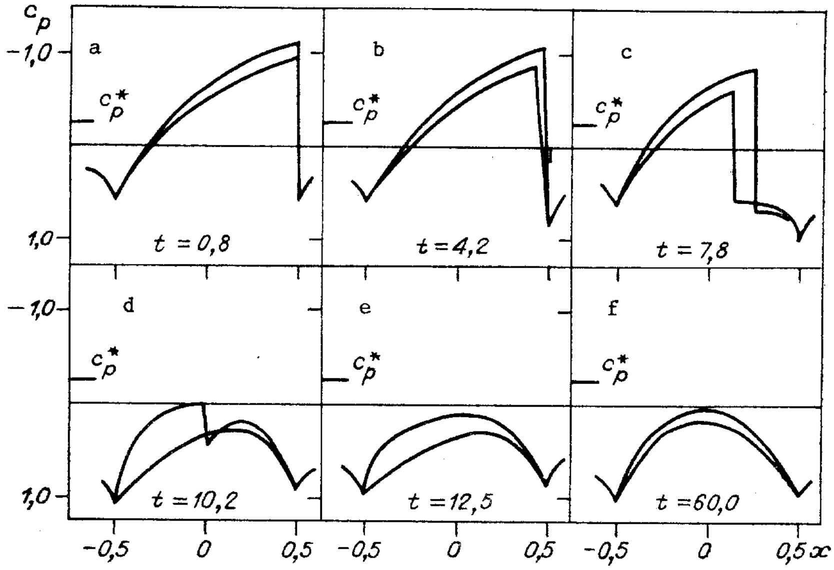

Fig. 4

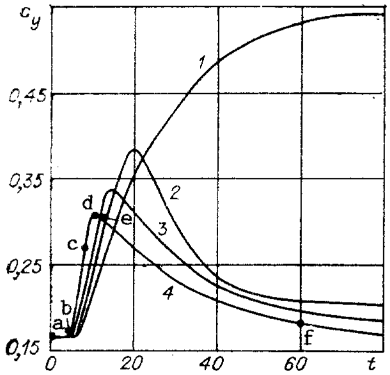

Fig. 5

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CALCULATION OF THE MOVEMENT OF A TWISTED FLOW OF A GAS SUSPENSION ABOUT THE END OF A SEMI-INFINITE CYLINDER

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This article examines the transverse movement of a twisted flow of a gas suspension about the end of a semi-infinite cylinder. The flow of the suspension is studied near the contact surface. The study is conducted within the framework of a three-velocity, three-temperature scheme describing the motion of interpenetrating continua. Questions relating to the formulation of the boundary conditions are also discussed. We determine the range of variation of the governing parameters within which reverse-circulating flow of the gas and particles takes place.

In most of the theoretical studies devoted either to the external flow of a gas suspension about a body or to the investigation of internal flows of disperse media, it is assumed particles which come into contact with a solid surface disappear from the flow [1-5]. Such a formulation of the problem is most appropriate for the case when the disperse phase consists of liquid drops or particles which form a thin film along the surface of the body after they come into contact with it.

If the disperse phase forms solid particles, the formulation of the boundary conditions becomes more complicated: it is necessary to introduce additional phases – a phase of particles reflected from the solid surface [6, 7] and a phase of particles moving randomly near the body in the flowing gas suspension [8].

1. Formulation of the Problem. We will examine the movement of a twisted flow of a gas suspension around a semi-infinite cylindrical end located within a contact surface which is coaxial with it (Fig. 1).

In accordance with [7], we introduce a fraction (phase) of incident particles (particles flying to the surface of the body in the flow) and a fraction of reflected particles (particles flying away from the surface, in the direction opposite the incident particles). As has already been noted, in the case of flow past blunt bodies, allowance for collisions between particles of different fractions makes it necessary to introduce an additional particle phase which moves randomly near the surface of the body in the gas suspension. Here, it is necessary to consider the velocity, pressure, and energy associated with the random motion resulting from collisions of particles of different fractions. Now the formulation of the problem is complicated to the extent that its cannot even be modeled numerically on a computer. Investigators have therefore found the range of determining parameters within which effects connected with randomization of the particles can be ignored. Thus, the estimates reported in [9] showed that randomization of the particles can be ignored when the mass content of particles in the incoming flow is on the order of 0.5-1. Within the framework of the proposed model, the equations describing the given problem have the form [7]

$$\frac{\partial\rho_1}{\partial t}+\operatorname{div}\rho_1\mathbf{v}_1=0,\quad \frac{\partial\rho_i}{\partial t}+\operatorname{div}\rho_i\mathbf{v}_i=J_{ij}\quad (i\neq j;\ i,\ j=2,\ 3),$$

$$\frac{\partial\rho_1\mathbf{v}_1}{\partial t}+\nabla^k\left(\rho_1\mathbf{v}_1v_1^k\right)=-\nabla p-\mathbf{f}_{12}-\mathbf{f}_{13},$$

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